Stability of coaxial skyrmion–vortex configurations due to ferroelectrical effects in ferromagnet–superconductor heterostructures

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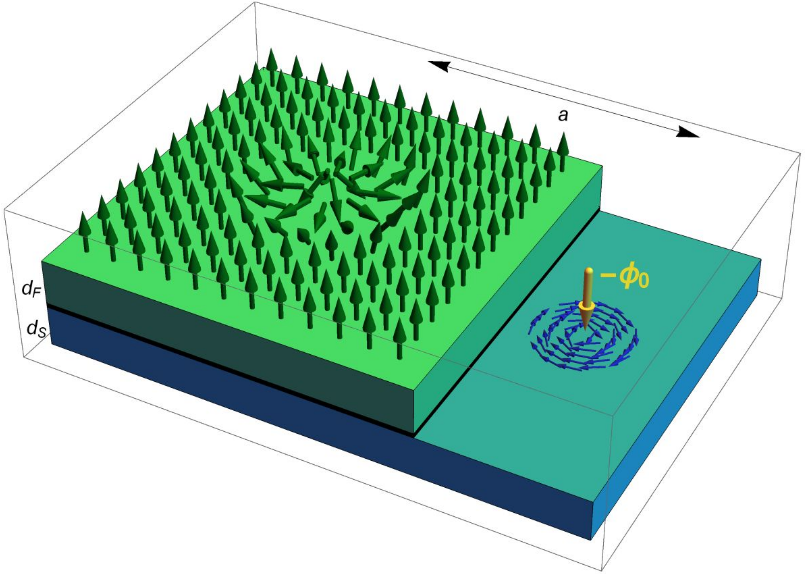
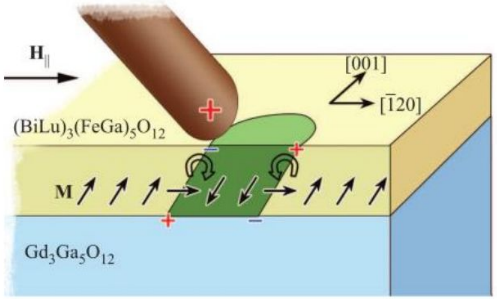
Introduction

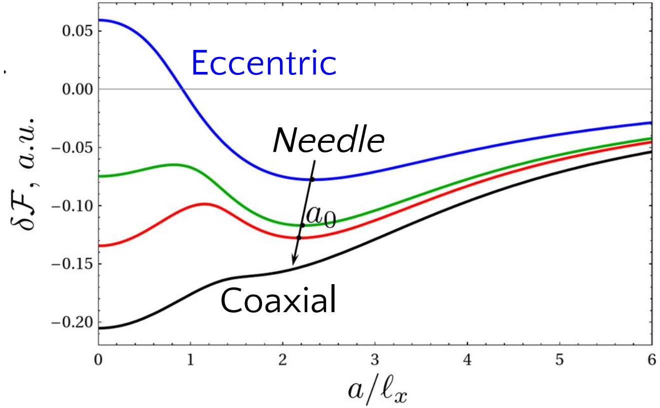
An eccentric configuration can be stable:



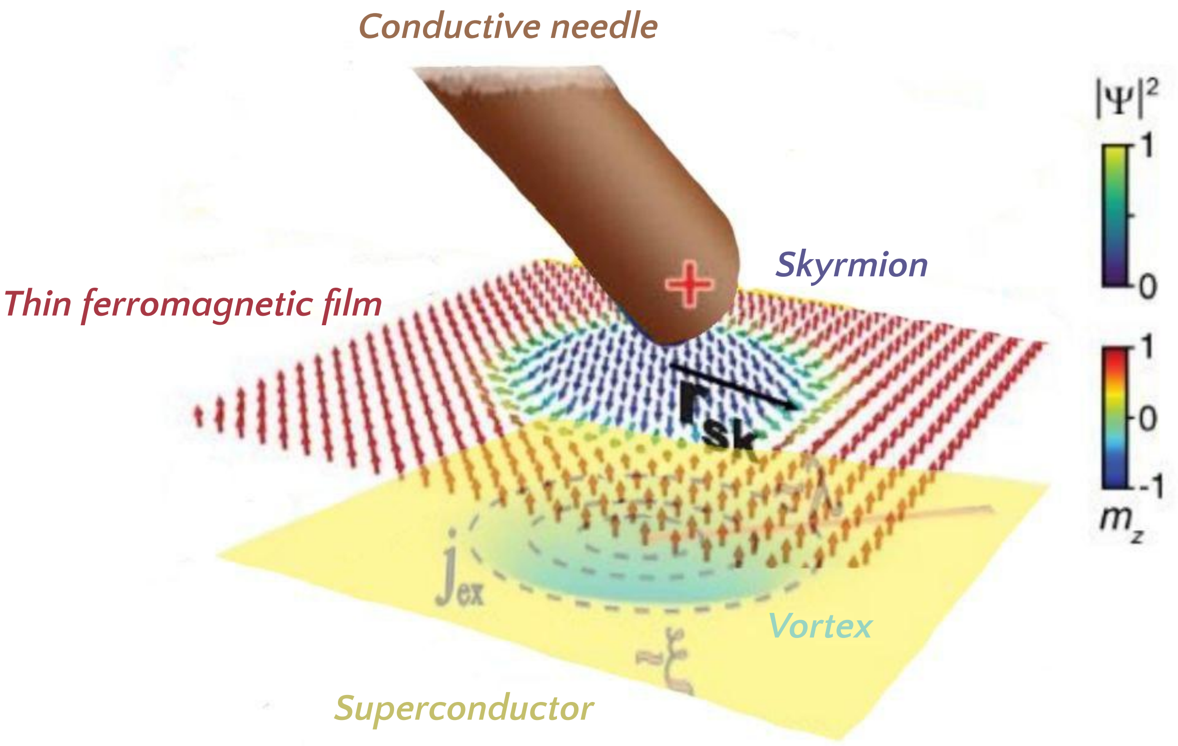
Majorana modes are predicted for coaxial state!



A.P. Pyatakov et al., Journal of Magnetism and Magnetic Materials 383, 255 (2015).



Problem setting

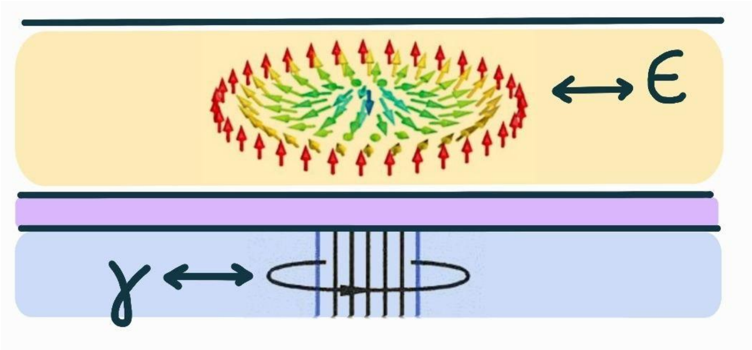


The base model

Ferromagnetic film and skyrmion

Insulator layer

Superconductor with vortex



$$\ell_w = \sqrt{A/K}$$

$$\epsilon = D/2\sqrt{AK}$$

$$\gamma = (\ell_w/\lambda)(M_s\phi_0/8\pi A)$$

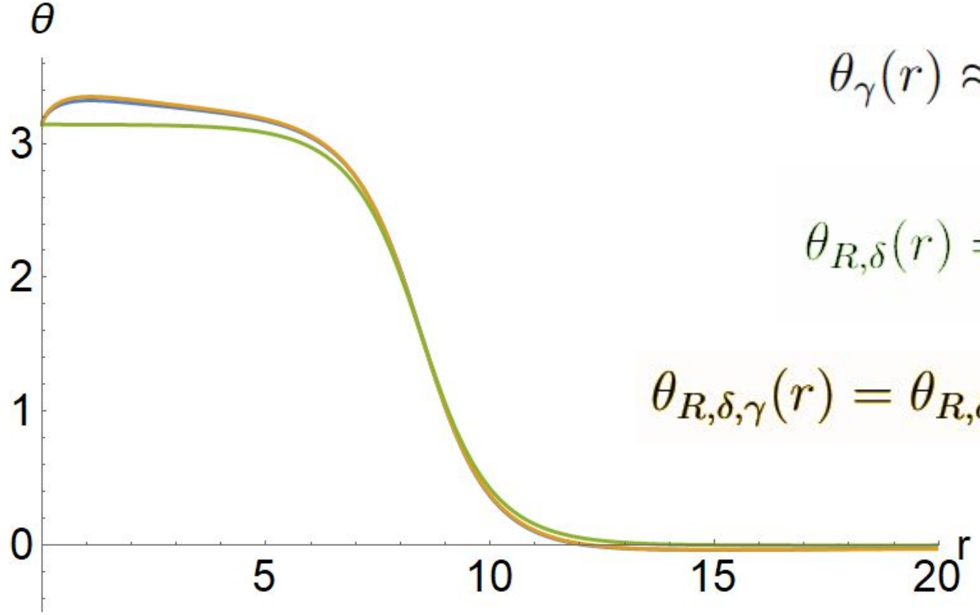
$$F[\mathbf{m}] = d_f \int d^2\mathbf{r}\left(A(\nabla\mathbf{m})^2 + K(1-m_z^2) + D[m_z\nabla\cdot\mathbf{m} - (\mathbf{m}\cdot\nabla)m_z] - M_s\mathbf{m}\cdot\mathbf{B}_v\right)$$

Exchange and anisotropy — *DMI* — *Zeeman energy*

$$F[\theta(r)] = 2\pi d_f \int dr\, r\left(A\left((\theta')^2 + \frac{\sin^2\theta}{r^2}\right) + K\sin^2\theta + D\left(\frac{\cos\theta\sin\theta}{r} + \theta'\right) - M_s(B_r\sin\theta + B_z\cos\theta)\right)$$

The Euler–Lagrange equation for the base model

$$2\left(\theta'' + \frac{\theta'}{r}\right) - \left(\frac{1}{r^2} + 1\right)\sin(2\theta) + \frac{4\epsilon\sin^2\theta}{r} + 2\gamma\left(\frac{\sin\theta}{r} - \frac{\cos\theta}{r}\right) = 0.$$



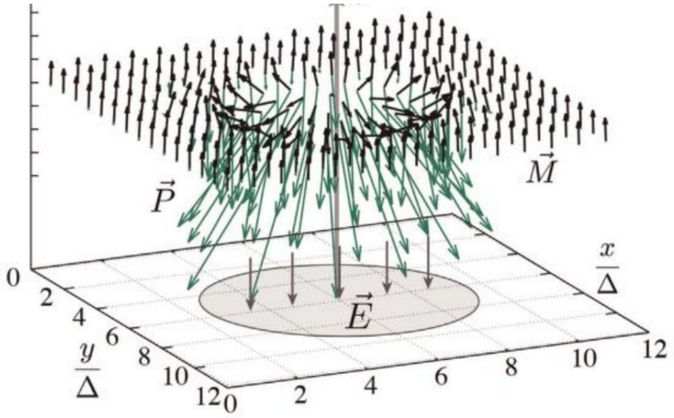
$$\theta_\gamma(r) \approx \gamma[K_1(r) - 1/r]$$

$$\theta_{R,\delta}(r) = 2\tan^{-1}\frac{\sinh\left(\frac{R}{\delta}\right)}{\sinh\left(\frac{r}{\delta}\right)}$$

$$\theta_{R,\delta,\gamma}(r) = \theta_{R,\delta}(r) + \theta_\gamma(r)\cos(\theta_{R,\delta}(r))$$

Polarization

1. Барьяхтар В.Г., Львов В.А., Яблонский Д.А., **Теория неоднородного магнитоэлектрического эффекта**, Письма в ЖЭТФ 37, 565 (1983).
2. M. Mostovoy, **Ferroelectricity in Spiral Magnets**, Phys. Rev. Lett. 96, 067601 (2006).
3. I. Dzyaloshinskii, **Magnetoelectricity in ferromagnets**, EPL 83, 67001 (2008).



A.P. Pyatakov et al., Journal of Magnetism and Magnetic Materials 383, 255 (2015).

$$\mathbf{P} = \alpha\chi_e M_s^2[(\mathbf{m}\cdot\nabla)\mathbf{m} - \mathbf{m}(\nabla\cdot\mathbf{m})]$$

For $\mathbf{m} = \mathbf{e}_r \sin\theta(r) + \mathbf{e}_z \cos\theta(r)$

$$P_r = -\alpha\chi_e M_s^2 \frac{\sin^2\theta}{r},$$

$$P_z = -\alpha\chi_e M_s^2 \left(\theta' + \frac{\sin\theta\cos\theta}{r}\right).$$

Ferroelectric effects

Interaction with the skyrmion adds $\int d^2\mathbf{r}\frac{4\pi\mathbf{P}\cdot\mathbf{E}}{2}$ to the free energy.

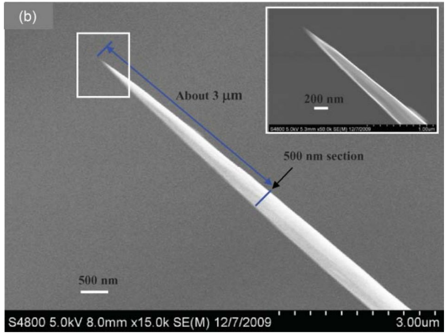
$$\Delta F[\theta] = -2\pi d_f \alpha \chi_e M_s^2 \int dr\, r E_z(r) \left(\theta' + \frac{\sin\theta\cos\theta}{r}\right).$$

Like DMI term!

The general Euler-Lagrange equation:

$$2\left(\theta'' + \frac{\theta'}{r}\right) - \left(\frac{1}{r^2}+1\right)\sin(2\theta) + \frac{4(\epsilon - \beta e_z)\sin^2\theta}{r} - 2\beta e_z' + 2\gamma\left(b_z\sin\theta - b_r\cos\theta\right) = 0.$$

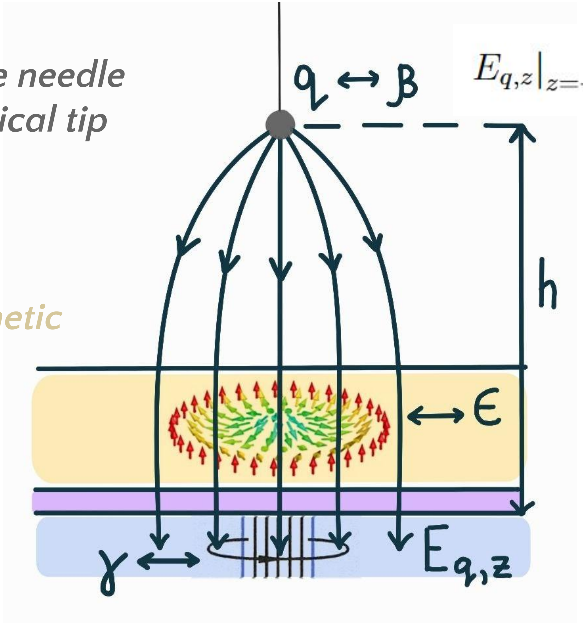
The needle modelling



Yuan-Liu Chen et al., Review of Scientific Instruments 82, 013707 (2011)

Conductive needle with spherical tip

Ferromagnetic film and skyrmion



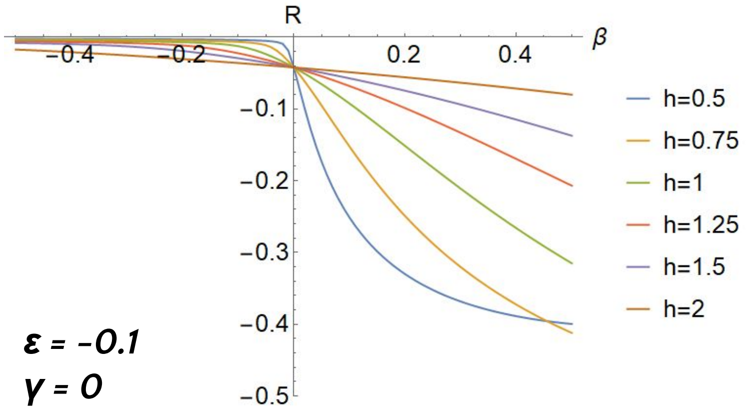
$$E_{q,z}|_{z=+0} = \frac{2qh}{(h^2+r^2)^{3/2}}.$$

Insulator layer

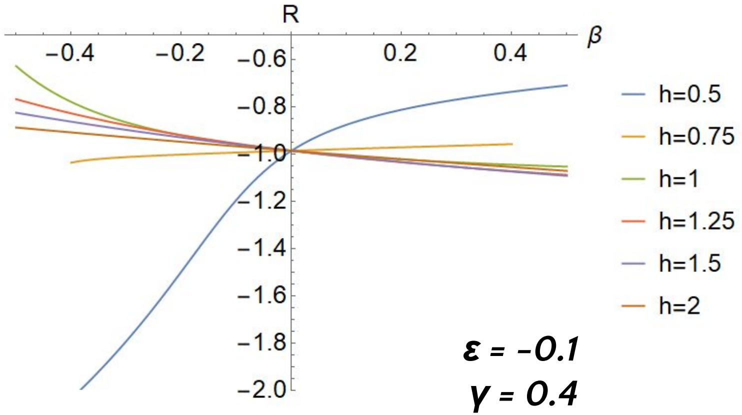
Superconductor with vortex

$$2\left(\theta'' + \frac{\theta'}{r}\right) - \left(\frac{1}{r^2}+1\right)\sin(2\theta) + 4\left(\epsilon - \frac{\beta}{(h^2+r^2)^{3/2}}\right)\frac{\sin^2\theta}{r} + 2\gamma\left(\frac{\sin\theta}{r} - \frac{\cos\theta}{r}\right) + \frac{6\beta r}{(r^2+h^2)^{5/2}} = 0$$

Size changing

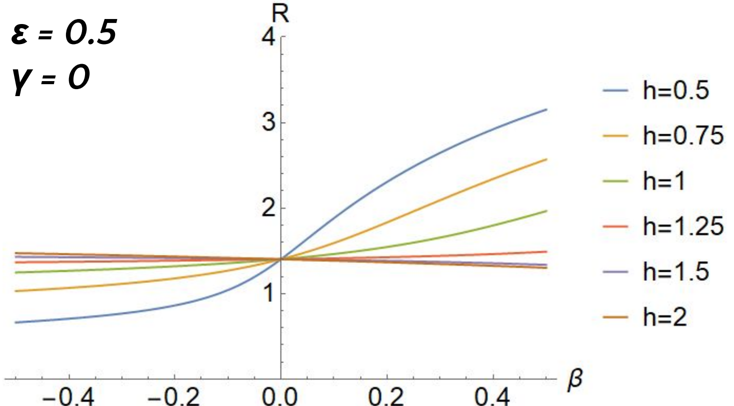


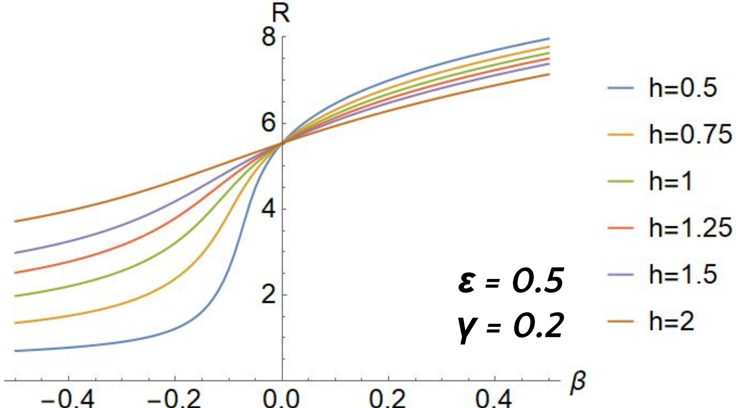
ε = -0.1
γ = 0



ε = -0.1
γ = 0.4

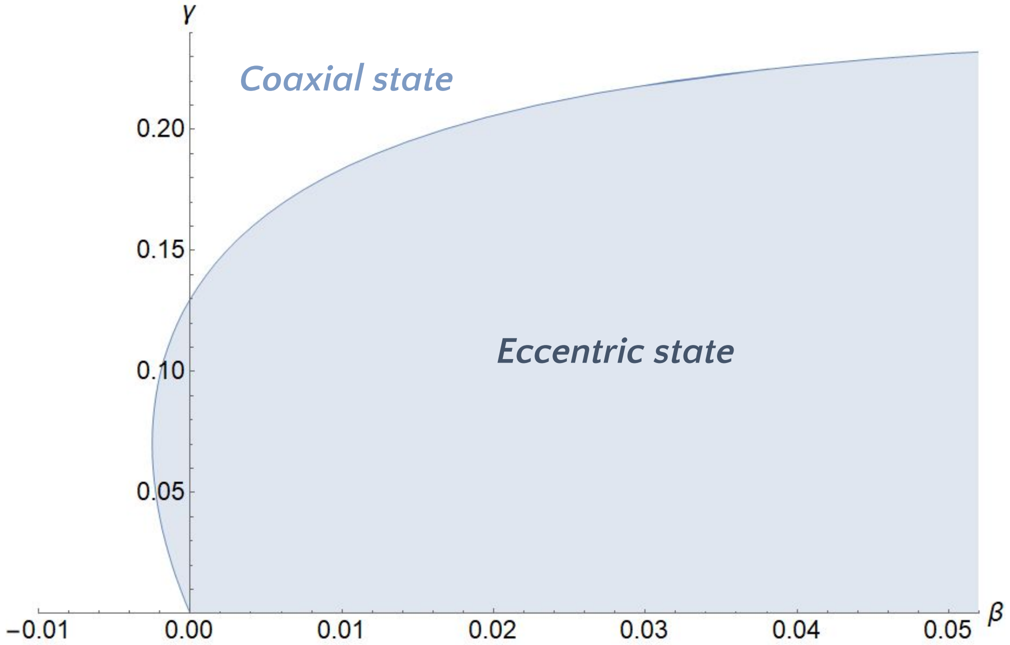
ε = 0.5
γ = 0





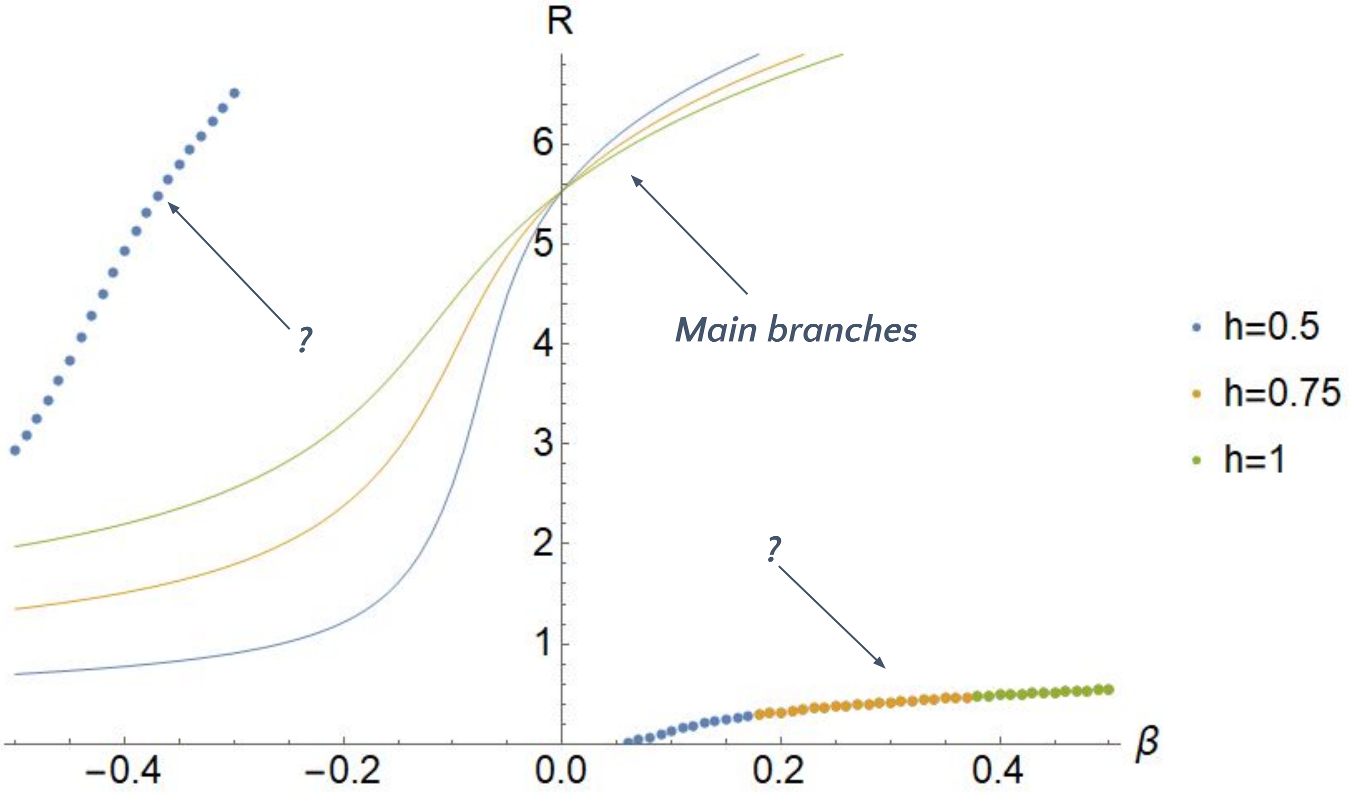
ε = 0.5
γ = 0.2

Stability of the coaxial configuration



$$\frac{\delta F(a)}{4\pi A d_f} = \int_0^\infty dr\{\gamma r[\delta b_r^a \sin\theta + \delta b_z^a(\cos\theta - 1)] - \beta\delta e_z^a\,(\sin\theta\cos\theta + r\theta')\}$$

Further research



Conclusion

- Reproduced well-known results for:
  - “No skyrmion” configuration
  - Free skyrmion
  - Skyrmion with vortex
- Obtained equations for the coaxial state of a skyrmion, vortex and a point charge
- Studied the change in size of a skyrmion and stability of the coaxial configuration due to the presence of a point charge

Appendix A.

$$\delta b_r^a = -\frac{\Theta(a-r)}{r}, \quad \delta b_z^a = K\left[\frac{4ar}{(a+r)^2}\right]\frac{2}{\pi(a+r)} - \frac{1}{r} \tag{15}$$

$$\delta e_z^a = \frac{2\sqrt{h^2+(a-r)^2}E\left(-\frac{4ar}{h^2+(-a+r)^2}\right)}{a^4+2a^2(h-r)(h+r)+\left(h^2+r^2\right)^2} + \\ + \frac{2\sqrt{h^2+(a+r)^2}E\left(\frac{4ar}{h^2+(a+r)^2}\right)}{a^4+2a^2(h-r)(h+r)+\left(h^2+r^2\right)^2} - \frac{1}{(r^2+h^2)^{3/2}} \tag{16}$$

Here, $\Theta(z)$ denotes the Heaviside step function, $K(z)$ and $E(z)$ are the complete elliptic integrals of the first and the second kind respectively.